Name: STANFORD Date: _____

NV College Physics Quiz: Calculus in Physics

Section 1: Antiderivatives. Find the antiderivative or definite integral of the following functions, show all work and circle or box your answer.

1. $y(x) = 6x^3 + 8x + 4$

$\int y(x)\,dx = \frac{3}{2}x^4 + 4x^2 + 4x + C$

2. $x(t) = 12t^5 - 8t^3$

$\int x(t)\,dt = 2x^6 - 2x^4 + C$

3. $y(x) = 2\sin x - 3\cos x$

$\int y(x)\,dx = -2\cos(x) - 3\sin(x) + C$

4. $x(t) = 6e^{-t}$

$\int x(t)\,dt = -6e^{-t} + C$

5. $y(x) = -3\sin(2x)$

$\int y(x)\,dx = \frac{3}{2}\cos(2x) + C$

6. $y(t) = 5e^{-2t}$

$\int y(t)\,dt = -\frac{5}{2}e^{-2t} + C$

7. $\int_0^3 (3x^2 + x - 2)\,dx$

$\left(x^3 + \frac{1}{2}x^2 - 2x\right)\Big|_0^3 = 3^3 + \frac{1}{2}(3^2) - 2(3) - 0$

$= 27 + \frac{9}{2} - 6$

$= 25\frac{1}{2}$

8. $\int_1^5 \frac{x+1}{x}\,dx = \int_1^5 \left(\frac{x}{x} + \frac{1}{x}\right)dx = \int_1^5 \left(1 + \frac{1}{x}\right)dx$

$= (x + \ln x)\Big|_1^5$

$= (5 + \ln|5|) - (1 + \ln|1|)$ ($\ln|1| = 0$)

$= 4 + \ln|5|$

Section 3: Calculus in Physics. Find the requested quantity or equation. Show all your work and circle or box your final answer. All values of position are given in meters, time is in seconds, velocity is in m/s.

1. The velocity of a particle is given by the equation $v(t) = 3t^2 - 4t + 3$

a) find the displacement of the particle between t = 2s and t = 4s

$$\Delta x = \int_2^4 (3t^2 - 4t + 3)\,dt = (t^3 - 2t^2 + 3t)\Big|_2^4 = 4^3 - 2(4^2) + 12$$

$$= 44\text{m}$$

b) find an equation for the position of the particle if x(0) = 2

$$x(t) = \int (3t^2 - 4t + 3)\,dt = t^3 - 2t^2 + 3t + C$$

$$0^3 - 2(0^2) + 3\cdot 0 + C = 2$$

$$C = 2 \quad \text{so} \quad x(t) = t^3 - 2t^2 + 3t + 2$$

c) Write an equation for the acceleration of the particle as a function of time.

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}(3t^2 - 4t + 3) = 6t - 4$$

2. The acceleration of a satellite is given as a(t) = 2t+1.

a) find an expression for the velocity as a function of time if the initial velocity at t=0 is 0.

$$v(t) = \int a(t)\,dt = \int (2t+1)\,dt = t^2 + t + C$$

$$0^2 + 0 + C = 0, \quad C = 0$$

$$v(t) = t^2 + t$$

b) Find the displacement between 0 and 5s.

$$\Delta x = \int_0^5 v(t)\,dt = \int_0^5 (t^2 + t)\,dt = \left(\frac{1}{3}t^3 + \frac{1}{2}t^2\right)\Big|_0^5 = \frac{125}{3} + \frac{25}{2} = 54\frac{1}{6}$$

c) Write an equation for the position of the satellite as a function of time if x(0) = 6.

$$x(t) = \int v(t)\,dt = \int (t^2 + t)\,dt = \frac{1}{3}t^3 + \frac{1}{2}t^2 + C$$

$$\frac{1}{3}(0) + \frac{1}{2}(0) + C = 6, \quad C = 6$$

$$x(t) = \frac{1}{3}t^3 + \frac{1}{2}t^2 + 6$$